PARLIAMENTARY VOTING RULES AND STRATEGIC CANDIDACY

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ABSTRACT. *In this paper we study the vulnerability of parliamentary voting procedures to strategic candidacy. Candidates involved in an election are susceptible to influence the outcome by opting out or opting in. In the context of three-alternative elections and under the impartial anonymous culture assumption, we evaluate the frequencies of such strategic candidacy opportunities.*

Keywords: *strategic candidacy, parliamentary voting procedures, opting out, opting in, Iimpartial anonymous culture.*
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1. INTRODUCTION

In democratic societies the outcomes of an election result from individual preferences defined on a set of alternatives. However, if the voting rule is based on sequential votes, then the outcomes can depend on the definition of the issue - that is the feasible set of alternatives or candidates. In other words, in most voting contexts, voting outcomes emerge from the set of actual candidates, and not from the set of all possibly potential candidates. We implicitly assume an election context in which voters' preferences are defined over the set of all potential candidates, but only votes on actual candidates are taken into account. From this set of potential candidates, we consider two subsets: the initial set of candidates and the set obtained after the entry of some other candidate or the exit of one candidate. *Strategic candidacy* occurs when some candidate has the opportunity to change the outcome of the voting rule in his favour by his simple entry or exit, given the preferences of the voters.

Strategic candidacy has been studied by several authors. For example, Osborne and Slivinski (1996), Besley and Coate (1997), Dutta, Jackson and Le Breton (2000, 2001), Eraslan and McLennan (2004), Samejima (2005), Samejima (2007) provide many results on this topic.

Although each of the above mentioned papers studies issues, incentives related to strategic candidacy or classes of voting rules immune to strategic candidacy, the focus of our paper is quite different. We study the vulnerability of parliamentary voting procedures to strategic candidacy. Specifically, we examine the vulnerability to strategic candidacy of amendment and successive elimination voting procedures (see details in Section 2), which are extensively used throughout the world for voting on motions in parliaments (see for example Rasch, 2000). More precisely, under these rules and in the context of three-alternative elections, our aim is to determine how frequent opportunities of this phenomenon are. We do this for the two versions of strategic candidacy mentioned above: opting out, and opting in.

The paper is organized as follows: in section 2 we introduce notations and definitions; Section 3 is devoted to the characterization of strategic candidacy possibilities. Then, Section 4 provides our results, and Section 5 concludes the paper.

2. Definitions and notations

2.1. **Candidates and voters.** Consider an election in which $A = \{a_1, a_2, a_3\}$ is a finite set of 3 alternatives or potential candidates, and N is the set of n individuals or voters, whose preferences are aggregated in order to determine the elected candidate. We also assume that candidates are allowed to vote, which is the case in many elections.

2.2. **Preference orders.** Over the set of candidates, individual preference relations are linear orders (complete, transitive and antisymmetric binary relations on A). Since $A = \{a_1, a_2, a_3\}$, the preference relation R^i of a given voter i is one of the following six possible linear orders over A: $R_1 : a_1a_2a_3$; $R_2 : a_1a_3a_2$; $R_3 : a_2a_1a_3$; $R_4 : a_2a_3a_1$; $R_5 : a_3a_1a_2$; $R_6 : a_3a_2a_1$.

The total number of voters whose preference relation is R_j ($j = 1, 2, ..., 6$) will be denoted n_j. Since n is the total number of voters, $n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = n$.

2.3. **Amendment voting procedure.** It consists in organizing a succession of qualified majority duels between the various alternatives. More precisely, it amounts to define an agenda - a predetermined order, say $a_1a_2a_3$ in this paper - between the various alternatives in the following way: a_1 against a_2, and the winner against a_3. The winner of this last confrontation is declared elected.

2.4. **Successive elimination voting procedure.** As for the above voting procedure, successive elimination is based on an agenda. However, at the first step a (possibly qualified) majority vote is organized on a_1, and if a_1 wins a majority, a_1 is elected and the procedure ends. If not, at the second step a vote is organized on a_2 in the same way, and

a_2 is elected if it collects a majority of votes. If neither a_1 nor a_2 collect a majority of votes, then a_3 is elected. Note that, when individuals vote on a1, they must in fact decide whether they prefer alternative a_1 to the subset $\{a_2, a_3\}$ or $\{a_2, a_3\}$ to a_1. In other words, they compare subsets of alternatives of possibly more than one element. Then, we distinguish two possible types of behavior: maximin (a pessimistic behavior), or maximax (an optimistic behavior).

2.4.1. *Maximin behavior.* Under maximin behavior, a voter does not vote for a candidate only when he ranks him at the last position in his preference order.

2.4.2. *Maximax behavior.* Under maximax behavior, a voter votes for a candidate when he ranks him at the first position in his preference order.

Under either voting procedure, if there is an equality between any two alternatives, ties are broken in favour of the one with the greatest index.

2.5. **Strategic candidacy.** It occurs when some candidate can exit (or enter) the election and change the outcome in his favour. In order to give some formal definitions of these notions, we need additional notations. Let F be voting procedure and let $R^N = (R^1, ..., R^n)$ denote a profile of individual preferences, one preference relation R^i for each individual i. Let $R^N|A - \{a_h\} = (R^1|A - \{a_h\}, ..., R^n|A - \{a_h\})$ denote the restriction of every individual preferences to the subset $A - \{a_h\}$ of alternatives.

2.5.1. *Opting out.* A voting procedure F is vulnerable to strategic candidacy at profile R^N by *opting out* if there exist $h, k, l \in \{1, 2, 3\}$ and some linear order R such that $a_h R x$ for all $x \neq a_h$, $a_k R a_l$, $F(R^N) = a_l$ and $F(R^N|A - \{a_h\}) = a_k$.

2.5.2. *Opting in.* A voting procedure F is vulnerable to strategic candidacy at profile R^N by *opting in* if there exist $h, k, l \in \{1, 2, 3\}$ and some linear order R such that $a_h R x$ for all $x \neq a_h$, $a_k R a_l$, $F(R^N|A - \{a_h\} = a_l$ and $F(R^N) = a_k$.

2.6. **Impartial anonymous culture.** Under impartial anonymous culture (IAC), voters are anonymous, in the sense that their identity does not matter : if we permute the preferences of two individuals, this will have no consequence on the outcome of the vote. Two profiles of preferences are considered as identical if in these two profiles the number of voters having the same type of preference relations is identical. Consequently in the sequel, instead of profiles in the sense defined above, we consider *voting situations* or simply *situations*, defined as vectors of the following form: $s = (n_1, n_2, n_3, n_4, n_5, n_6)$. Nonetheless and for convenience, all examples will still be given in terms of profiles.

3. Strategic candidacy possibilities

In this section we characterize all voting situations at which there exists an opportunity for strategic candidacy. We successively study opting out and opting in. We begin with opting out.

3.1. **Opting out.** We first consider the amendment procedure.

3.1.1. *Amendment voting procedure (AmP).*

a_1 is elected. This case occurs only if a_1 is a *Condorcet winner* (CW), that is a_1 beats a_2, and a_1 beats a_3. Then clearly, there is no way for strategic candidacy.
a_2 is elected. As above, this occurs only if a_2 is a Condorcet winner, that is a_2 beats a_1, and a_2 beats a_3. Then again, there is no way for strategic candidacy.
a_3 is elected. This case occurs only under two possibilities:
(i) a_1 opts out.

Example 1. *Consider the following profile:*

1	*2*	*3*		*1*	*2*	*3*
a_1	a_2	a_3	$\Longrightarrow$	a_2	a_2	a_3
a_2	a_3	a_1		a_3	a_3	a_2
a_3	a_1	a_2				

a_1 beats a_2, a_3 beats a_1; then a_3 wins. If a_1 opts out, a_2 beats a_3 and it follows that a_2 wins. We then conclude that a_1 is a *strategic candidate*.

Note that in Example 1 there is a *Condorcet cycle* (Type 1): a_1 beats a_2, a_3 beats a_1 and a_2 beats a_3. More generally, *AmP* is vulnerable to strategic candidacy at profile R^N if there exists a Condorcet cycle

(Type 1), which is equivalent to the following inequalities:

$$\begin{cases} a_1 \text{ beats } a_2 \\ a_3 \text{ beats } a_1 \\ a_2 \text{ beats } a_3 \end{cases} \Leftrightarrow \begin{cases} n_1+n_2+n_5 > n-\alpha n \\ n_4+n_5+n_6 \geq \alpha n \\ n_1+n_3+n_4 > n-\alpha n \\ n_3+n_4 \geq 1 \\ n_5+n_6 \geq 1 \\ n_1 \geq 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} S_1 \\ n_3+n_4 \geq 1 \\ n_5+n_6 \geq 1 \\ n_1 \geq 1 \end{cases}$$

(ii) a_2 opts out.

Example 2. *Consider the following profile:*

1	*2*	*3*		*1*	*2*	*3*
a_1	a_2	a_3	$\Longrightarrow$	a_1	a_1	a_3
a_3	a_1	a_2		a_3	a_3	a_2
a_2	a_3	a_1				

a_2 beats a_1, a_3 beats a_2 and then a_3 wins. If a_2 opts out, a_1 beats a_3 and then a_1 wins. It follows that a_2 is a strategic candidate.

More generally AmP is vulnerable to strategic candidacy at R^N if there exists a Condorcet cycle (Type 2) : a_2 beats a_1, a_3 beats a_2 and a_2 beats a_1, which is equivalent to the following inequalities:

$$\begin{cases} a_2 \text{ beats } a_1 \\ a_3 \text{ beats } a_2 \\ a_1 \text{ beats } a_3 \end{cases} \Leftrightarrow \begin{cases} n_3+n_4+n_6 \geq \alpha n \\ n_2+n_5+n_6 \geq \alpha n \\ n_1+n_2+n_3 > n-\alpha n \\ n_1+n_2 \geq 1 \\ n_5+n_6 \geq 1 \\ n_3 \geq 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} S_2 \\ n_1+n_2 \geq 1 \\ n_5+n_6 \geq 1 \\ n_3 \geq 1 \end{cases}$$

It is clear that when a_3 opts out, there is no incentive for strategic candidacy. Indeed, candidate a_3 who is already elected, does not way find it beneficial to exit since he cannot hope to obtain a better situation. The proposition below summaries all these observations.

Proposition 1. *Let AmP be the amendment voting procedure and $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy iff*

$$\begin{cases} S_1 \\ n_3 + n_4 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_1 \geq 1 \end{cases} \quad or \quad \begin{cases} S_2 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{cases}$$

We next study successive elimination under maximin and maximax behavior.

3.1.2. *Successive elimination voting procedure: maximin behavior* ($SE\min$).
- a_1 is elected. As above, we distinguish two different possibilities:
(i) a_2 opts out.

Example 3. *Consider the following profile*

1	***2***	*3*
a_1	a_2	a_3
a_2	a_3	a_1
a_3	a_1	a_2

$\Longrightarrow$

1	***2***	*3*
a_1	a_3	a_3
a_3	a_1	a_1

a_1 beats $\{a_2 a_3\}$, then a_1 wins. If a_2 opts out, a_3 beats a_1 and then a_3 wins. a_2 is a strategic candidate.

More generally $SE\min$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_2 a_3 a_1$, which is equivalent to the following inequalities:

$$\begin{cases} a_1 \text{ beats } \{a_2, a_3\} \\ a_3 \text{ beats } a_1 \\ |\{i \in N : R^i = a_2 a_3 a_1\}| \geq 1 \end{cases} \Leftrightarrow \begin{cases} n_1 + n_2 + n_3 + n_5 > n - \alpha n \\ n_4 + n_5 + n_6 \geq \alpha n \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} S_3 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{cases}$$

(ii) a_3 opts out.

Example 4. *Consider the following profile*

1	*2*	***3***
a_1	a_2	a_3
a_2	a_1	a_2
a_3	a_3	a_1

$\Longrightarrow$

1	*2*	***3***
a_1	a_2	a_2
a_2	a_1	a_1

a_1 beats $\{a_2 a_3\}$, then a_1 wins. If a_3 opts out, a_2 beats a_1 and then a_2 wins. a_3 is a strategic candidate.

More generally $SE\min$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_3 a_2 a_1$, which is equivalent to the following inequalities:

$$\left\{\begin{array}{c} a_1 \text{ beats } \{a_2, a_3\} \\ a_2 \text{ beats } a_1 \\ |\{i \in N : R^i = a_3 a_2 a_1\}| \geq 1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{c} n_1 + n_2 + n_3 + n_5 > n - \alpha n \\ n_3 + n_4 + n_6 \geq \alpha n \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_6 \geq 1 \end{array}\right.$$

$$\Leftrightarrow \left\{\begin{array}{c} S_4 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_6 \geq 1 \end{array}\right.$$

- a_2 is elected. There is no way for strategic candidacy. Indeed, when candidates a_2 is elected there must be that a_1 is eliminated at the first step, and that a_2 beats a_3: and this means that even if a_1 or a_3 exits, candidate a_2 will still win.
- a_3 is elected. Again, there is no way for strategic candidacy, for very similar reasons as above.

Proposition 2. *Let $SE\min$ be the Successive elimination voting procedure under maximin and $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy iff*

$$\left\{\begin{array}{c} S_3 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{array}\right. \quad or \quad \left\{\begin{array}{c} S_4 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_6 \geq 1 \end{array}\right.$$

3.1.3. *Successive elimination voting procedure: maximax behavior* ($SE\max$).
- a_1 is elected. No strategic candidacy. The same arguments as above apply.
- a_2 is elected.
(i) a_1 opts out. No strategic candidacy.
(ii) a_2 opts out. Again, no way for strategic candidacy.
(iii) a_3 opts out.

Example 5. *Consider the following profile*

1	*2*	***3***
a_1	a_2	a_3
a_2	a_1	a_1
a_3	a_3	a_2

$\Longrightarrow$

1	*2*	***3***
a_1	a_2	a_1
a_2	a_1	a_2

$\{a_2a_3\}$ beats a_1, a_2 beats a_3, then a_2 wins. If a_3 opts out, a_1 beats a_2 and a_1 wins. a_3 is a strategic candidate.

More generally $SE\max$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_3a_1a_2$, which is equivalent to the following inequalities:

$$\left\{\begin{array}{c} \{a_2, a_3\} \text{ beats } a_1 \\ a_2 \text{ beats } a_3 \\ a_1 \text{ beats } a_2 \\ |\{i \in N : R^i = a_3a_1a_2\}| \geq 1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{c} n_3 + n_4 + n_5 + n_6 \geq \alpha n \\ n_1 + n_3 + n_4 > n - \alpha n \\ n_1 + n_2 + n_5 > n - \alpha n \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \end{array}\right.$$

$$\Leftrightarrow \left\{\begin{array}{c} S_5 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \end{array}\right.$$

- a_3 is elected.
(i) a_1 opts out. No strategic candidacy. Indeed, when the candidate a_3 is elected; that suppose that a_1 is not mainly classified first and that a_3 beats a_2: What means that even if a_1 exits, the candidate a_3 will continue to win.
(ii) a_2 opts out.

Example 6. *Consider the following profile*

1	*2*	*3*		*1*	*2*	*3*
a_1	a_2	a_3	$\Longrightarrow$	a_1	a_1	a_3
a_3	a_1	a_1		a_3	a_3	a_1
a_2	a_3	a_2				

$\{a_2a_3\}$ beats a_1, a_3 beats a_2, then a_3 wins. If a_2 opts out, a_1 beats a_3, and a_1 wins. a_2 is a strategic candidate.

More generally $SE\max$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_2a_1a_3$, which is equivalent to the following inequalities:

$$\begin{cases} \{a_2, a_3\} \text{ beats } a_1 \\ a_3 \text{ beats } a_2 \\ a_1 \text{ beats } a_3 \\ |\{i \in N : R^i = a_2a_1a_3\}| \geq 1 \end{cases} \Leftrightarrow \begin{cases} n_3 + n_4 + n_5 + n_6 \geq \alpha n \\ n_2 + n_5 + n_6 \geq \alpha n \\ n_1 + n_2 + n_3 > n - \alpha n \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} S_6 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{cases}$$

(iii) a_3 opts out. No strategic candidacy. indeed, the candidate a_3 who is already elected, does not way find it beneficial to exit bus which cannot hope to obtain a better situation.

Proposition 3. *Let $SE\max$ be the Successive elimination voting procedure under maximax and $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy iff*

$$\begin{cases} S_6 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{cases} \quad or \quad \begin{cases} S_5 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \end{cases}$$

3.2. **Opting in.**

3.2.1. *Amendment voting procedure (AmP).*
- a_1 is elected.
(i) a_2 opts in. We have two possibilities:

(1) a_2 is a CW, a_2 beats a_1, and a_2 beats a_3. Then clearly, there is no way for strategic candidacy.

(2) a_2 has preference $a_2a_3a_1$ and is not a CW, and thus does not in any case win the election; however, if he does not run the election, a_1 wins, and if he runs, a_3 wins.

Example 7. *Consider the following profile*

1	*2*	*3*	*4*	*5*
a_3	a_1	a_3	a_1	a_1
a_1	a_3	a_1	a_3	a_3

$\Longrightarrow$

1	*2*	*3*	*4*	*5*
a_2	a_2	a_3	a_1	a_1
a_3	a_1	a_2	a_3	a_3
a_1	a_3	a_1	a_2	a_2

a_1 beats a_3, then a_1 wins. If a_2 opts in, a_2 beats a_1, a_3 beats a_2, then a_3 wins. a_2 is a strategic candidate.

More generally AmP is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_2a_3a_1$, which is equivalent to the following inequalities:

$$\begin{cases} a_2 \text{ beats } a_1 \\ a_1 \text{ beats } a_3 \\ a_3 \text{ beats } a_2 \end{cases} \Leftrightarrow \begin{cases} S_2 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{cases}$$

(iii) a_3 opts in. No possibility for strategic candidacy. Because when a_3 opts in, a_3 beats a_1 and is elected or a_1 beats a_3 and still wins.
- a_2 is elected.
(i) a_1 opts in. We have two possibilities:

(1) a_1 is a CW, a_1 beats a_2, and a_1 beats a_3. Then clearly, there is no way for strategic candidacy.

(2) a_1 has preference $a_1a_3a_2$ and is not a CW, and thus does not in any case win the election; however, if he does not run the election, a_2 wins, and if he runs, a_3 wins.

Example 8. *Consider the following profile*

1	*2*	*3*	*4*	*5*
a_3	a_2	a_3	a_2	a_2
a_2	a_3	a_2	a_3	a_3

$\Longrightarrow$

1	*2*	*3*	*4*	*5*
a_1	a_1	a_3	a_2	a_2
a_3	a_2	a_1	a_3	a_3
a_2	a_3	a_2	a_1	a_1

a_2 beats a_3 then a_2 wins. If a_1 opts in, a_1 beats a_2, a_3 beats a_1, then a_3 wins. a_1 is a strategic candidate**.**

More generally AmP is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_1a_3a_2$, which is equivalent to the following inequalities:

$$\begin{cases} a_1 \text{ beats } a_2 \\ a_3 \text{ beats } a_1 \\ a_2 \text{ beats } a_3 \end{cases} \Leftrightarrow \begin{cases} S_1 \\ n_3 + n_4 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_2 \geq 1 \end{cases}$$

(ii) a_3 opts in. No possibility for strategic candidacy. Because when a_3 opts in, a_3 beats a_2 and is elected or a_2 beats a_3 and continue to win.
- a_3 is elected. No strategic candidacy. Because when a_1 or a_2 opts in, a_1 or a_2 beats a_3 and is elected or a_3 beats a_1 or a_2 and continue to win.

Proposition 4. *Let AmP be the amendment voting procedure and $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy iff*

$$\left\{ \begin{array}{c} S_1 \\ n_3 + n_4 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_2 \geq 1 \end{array} \right. \quad or \quad \left\{ \begin{array}{c} S_2 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{array} \right.$$

3.2.2. *Successive elimination voting procedure: maximin behavior* ($SE\,\text{min}$).
- a_1 is elected. No strategic candidacy. Because a_1 is elected when it is not mainly classified in the last position. Thus, even if a_2 or a_3 opts in, a_1 always continue to win.
- a_2 is elected.
(i) a_1 opts in. No possibility for strategic candidacy. Because when a_1 opts in, a_1 beats $\{a_2a_3\}$ and is elected or $\{a_2a_3\}$ beats a_1 and a_2 continue to win.
(ii) a_3 opts in.

Example 9. *Consider the following profile*

$$\begin{array}{ccccc} \textbf{\textit{1}} & \textit{2} & \textit{3} & \textit{4} & \textit{5} \\ \hline a_1 & a_1 & a_2 & a_2 & a_2 \\ a_2 & a_2 & a_1 & a_1 & a_1 \end{array} \quad \Longrightarrow \quad \begin{array}{ccccc} \textbf{\textit{1}} & \textit{2} & \textit{3} & \textit{4} & \textit{5} \\ \hline a_3 & a_1 & a_2 & a_2 & a_2 \\ a_1 & a_2 & a_1 & a_1 & a_3 \\ a_2 & a_3 & a_3 & a_3 & a_1 \end{array}$$

a_2 beats a_1 then a_2 wins. If a_3 opts in, a_1 beats $\{a_2a_3\}$ then a_1 wins. a_3 is a strategic candidate.

More generally $SE\,\text{min}$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_3a_1a_2$, which is equivalent to the following inequalities:

$$\left\{ \begin{array}{c} a_2 \text{ beats } a_1 \\ a_1 \text{ beats } \{a_2, a_3\} \\ |\{i \in N : R^i = a_3a_1a_2\}| \geq 1 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{c} S_4 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \end{array} \right.$$

- a_3 is elected.
(i) a_1 opts in. No possibility for strategic candidacy. Because when a_1 opts in, a_1 beats $\{a_2a_3\}$ and is elected or $\{a_2a_3\}$ beats a_1 and a_3 continue to win.
(ii) a_2 opts in.

Example 10. *Consider the following profile*

1	*2*	*3*	*4*	*5*
a_1	a_3	a_1	a_3	a_3
a_3	a_1	a_3	a_1	a_1

$\Longrightarrow$

1	*2*	*3*	*4*	*5*
a_2	a_3	a_1	a_3	a_3
a_1	a_1	a_3	a_1	a_2
a_3	a_2	a_2	a_2	a_1

a_3 beats a_1 then a_3 wins. If a_2 opts in, a_1 beats $\{a_2a_3\}$, then a_1 wins. a_2 is a strategic candidate.

More generally $SE\min$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_2a_1a_3$, which is equivalent to the following inequalities:

$$\left\{\begin{array}{c} a_3 \text{ bat } a_1 \\ a_1 \text{ bat } \{a_2, a_3\} \\ |\{i \in N : R^i = a_2a_1a_3\}| \geq 1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{c} S_3 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{array}\right.$$

Proposition 5. *Let* $SE\min$ *be the Successive elimination voting procedure under maximin and* $0 < \alpha \leq 1$*, then a voting situation* s *is vulnerable to strategic candidacy iff*

$$\left\{\begin{array}{c} S_3 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{array}\right. \text{ or } \left\{\begin{array}{c} S_4 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \end{array}\right.$$

3.2.3. *Successive elimination voting procedure: maximax behavior* ($SE\max$).
- a_1 is elected.
(i) a_2 opts in.

Example 11. *Consider the following profile*

1	*2*	*3*	*4*	**5**
a_1	a_1	a_3	a_1	a_3
a_3	a_3	a_1	a_3	a_1

$\Longrightarrow$

1	*2*	*3*	*4*	**5**
a_2	a_1	a_3	a_1	a_2
a_1	a_3	a_2	a_3	a_3
a_3	a_2	a_1	a_2	a_1

a_1 beats a_3 then a_1 wins. If a_2 opts in, $\{a_2a_3\}$ beats a_1, a_3 beats a_2, then a_3 wins. a_2 is a strategic candidate.

More generally $SE\max$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_2a_3a_1$, which is equivalent to the following inequalities:

$$\begin{cases} a_1 \text{ beats } a_3 \\ \{a_2, a_3\} \text{ beats } a_1 \\ a_3 \text{ beats } a_2 \\ |\{i \in N : R^i = a_2a_3a_1\}| \geq 1 \end{cases} \Leftrightarrow \begin{cases} S_6 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{cases}$$

(ii) a_3 opts in.

Example 12. *Consider the following profile*

1	*2*	*3*	*4*	**5**
a_1	a_1	a_2	a_1	a_2
a_2	a_2	a_1	a_2	a_1

$\Longrightarrow$

1	*2*	*3*	*4*	**5**
a_3	a_1	a_2	a_1	a_3
a_1	a_2	a_3	a_2	a_2
a_2	a_3	a_1	a_3	a_1

a_1 beats a_2 then a_1 wins. If a_3 opts in, $\{a_2a_3\}$ beats a_1, a_2 beats a_3, then a_2 wins. a_3 is a strategic candidate**.**

More generally $SE\max$ is vulnerable to strategic candidacy at R^N if there exists some i such that $R^i = a_3a_2a_1$, which is equivalent to the following inequalities:

$$\begin{cases} a_1 \text{ beats } a_2 \\ \{a_2, a_3\} \text{ beats } a_1 \\ a_2 \text{ beats } a_3 \\ |\{i \in N : R^i = a_3a_2a_1\}| \geq 1 \end{cases} \Leftrightarrow \begin{cases} S_5 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_6 \geq 1 \end{cases}$$

- a_2 is elected. No strategic candidacy. Because when a_1 or a_3 opts in, a_1 or a_3 beats a_2 and is elected or a_2 beats a_1 or a_3 and continue to win.
- a_3 is elected. No strategic candidacy. Because when a_1 or a_2 opts in, a_1 or a_2 beats a_3 and is elected or a_3 beats a_1 or a_2 and continue to win.

Proposition 6. *Let $SE\max$ be the Successive elimination voting procedure under maximax and $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy iff*

$$\begin{cases} S_6 \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_4 \geq 1 \end{cases} \text{or} \begin{cases} S_5 \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_6 \geq 1 \end{cases}$$

4. Evaluation of strategic candidacy frequencies

Strategic candidacy frequencies will be calculated on the basis of this following ratio:

$$\frac{\text{Number of voting situations at which strategic candidacy is possible}}{\text{Total number of voting situations}}$$

The method used to compute these frequencies is based on Gehrlein and Fishburn (1976) and is the same as the one in Mbih, Moyouwou and Picot (2008). All technical details are available from the authors upon simple request.

Subsequently, one poses

$$p = \lfloor n - \alpha n \rfloor = \lfloor (1-\alpha)n \rfloor$$

p is the least integer less than or equal to $\lfloor (1-\alpha)n \rfloor$ and we obtain:

$$\begin{aligned} (4.1)\quad \sum_{j\in I} n_j \quad &> \quad n - \alpha n \Longleftrightarrow \sum_{j\in I} n_j \geq \lfloor (1-\alpha)n \rfloor + 1 = p+1 \\ &\Longleftrightarrow \quad \sum_{j\in I} n_j \geq p+1 \Longleftrightarrow -(\sum_{j\in I} n_j) \leq -p-1 \\ &\Longleftrightarrow \quad -(\sum_{j\in I} n_j) + p + 1 \leq 0 \end{aligned}$$

$$\begin{aligned} (4.2)\quad & \sum_{j\in I} n_j \geq \alpha n \Longleftrightarrow -(\sum_{j\in I} n_j) \leq -\alpha n \Longrightarrow n - (\sum_{j\in I} n_j) \leq n - \alpha n \\ & \Longleftrightarrow n - (\textstyle\sum_{j\in I} n_j) \leq [n(1-\alpha)] \ \Longleftrightarrow n - (\textstyle\sum_{j\in I} n_j) \leq p \quad (a) \\ & \text{Or } n - \textstyle\sum_{j\in I} n_j = \textstyle\sum_{j\in J} n_j \ \text{ where } \quad J = \{1, ..., 6\} - I \quad (b) \\ & (a) \text{ and } (b) \Longrightarrow \textstyle\sum_{j\in J} n_j \leq p \Longleftrightarrow \textstyle\sum_{j\in J} n_j - p \leq 0 \end{aligned}$$

$$(4.3)\qquad \sum_{j\in I} n_j \geq 1 \Longleftrightarrow -(\sum_{j\in I} n_j) \leq -1 \Longleftrightarrow -(\sum_{j\in I} n_j) + 1 \leq 0$$

4.1. **Opting out.**

4.1.1. *Amendment voting procedure.*

Proposition 7. *Let AmP be the amendment voting procedure and consider α such that $0 < \alpha \leq 1$, then a voting situation s is vulnerable to* *strategic candidacy by opting out if and only if*

$$\left\{\begin{array}{l} n_1+n_2+n_5>n-\alpha n \\ n_4+n_5+n_6\geq\alpha n \\ n_1+n_3+n_4>n-\alpha n \\ n_3+n_4\geq 1 \\ n_5+n_6\geq 1 \\ n_1\geq 1 \end{array}\right. \quad or \quad \left\{\begin{array}{l} n_3+n_4+n_6\geq\alpha n \\ n_2+n_5+n_6\geq\alpha n \\ n_1+n_2+n_3>n-\alpha n \\ n_1+n_2\geq 1 \\ n_5+n_6\geq 1 \\ n_3\geq 1 \end{array}\right.$$

Proposition 8. *Let $f(AmP,n,p)$ be the total number of voting situations at which AmP is vulnerable to strategic candidacy. Computations give:*

For $0<p\leq\frac{n}{2}$,

$$f(AmP,n,p)=\frac{\left(5n-p-35np+10n^2+32p^2+9\right)(p+2)(p+1)p}{120}$$

For $\frac{n}{2}\leq p<n$

$$f(AmP,n,p)=\frac{\left(203p-73n-59np+12n^2+7n^3+87p^2-32p^3+61np^2-36n^2p+102\right)(p-n+1)(p-n)}{120}$$

As a consequence,

Proposition 9. *The vulnerability $F(AmP,n,p)$ of the amendment voting rule to strategic candidacy by opting out is as follows:*
For $0<p\leq\frac{n}{2}$

$$F(AmP,n,p)=\frac{\left(5n-p-35np+10n^2+32p^2+9\right)(p+2)(p+1)p}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

For $\frac{n}{2}\leq p<n$

$$F(AmP,n,p)=\frac{\left(203p-73n-59np+12n^2+7n^3+87p^2-32p^3+61np^2-36n^2p+102\right)(p-n+1)(p-n)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

Proposition 10. *When n tends to infinity, the vulnerability $F'(AmP,\alpha)$ of the amendment voting rule to strategic candidacy by opting out is as follows:*

$$F'(AmP,\alpha)=\left\{\begin{array}{l} \alpha^3\left(32\alpha^2-35\alpha+10\right)\ if\ 0\leq\alpha\leq\frac{1}{2} \\ \left(32\alpha^2-29\alpha+7\right)(1-\alpha)^3\ \ if\ \frac{1}{2}\leq\alpha\leq 1 \end{array}\right.$$

Proposition 11. *When $\alpha=\frac{1}{2}$, the vulnerability $F''(AmP,n)$ of the amendment voting rule to strategic candidacy by opting out is as follows:*

$$F''(AmP,n)=\left\{\begin{array}{l} \frac{10n+n^2+21}{160n+16n^2-384}\ (if\ n\ is\ odd) \\ \frac{408n+114n^2+17n^3+n^4}{944n^2-272n+272n^3+16n^4-960}\ (if\ n\ is\ even) \end{array}\right.$$

4.1.2. *Successive elimination voting procedure: maximin behavior.*

Proposition 12. *Let* $SE\min$ *be the successive elimination voting procedure under maximin and consider* α *such that* $0 < \alpha \leq 1$*, then a voting situation* s *is vulnerable to strategic candidacy by opting out if and only if*

$$\left\{\begin{array}{l} n_1+n_2+n_3+n_5 > n-\alpha n \\ n_4+n_5+n_6 \geq \alpha n \\ n_1+n_2 \geq 1 \\ n_5+n_6 \geq 1 \\ n_4 \geq 1 \end{array}\right. \quad or \quad \left\{\begin{array}{l} n_1+n_2+n_3+n_5 > n-\alpha n \\ n_1+n_2+n_3 > n-\alpha n \\ n_3+n_4+n_6 \geq \alpha n \\ n_1+n_2 \geq 1 \\ n_3+n_4 \geq 1 \\ n_6 \geq 1 \end{array}\right.$$

Proposition 13. *Let* $f(SE\min, n, \alpha)$ *be the total number of voting situations at which AmP is vulnerable to strategic candidacy. Computations give:*

For $0 < p \leq \frac{n}{2}$

$$f(SE\min,n,p)=\frac{\left(76p-70n-245np+100n^2+154p^2+26p^3-45np^2+20n^2p-16\right)(p+1)p}{120}$$

For $\frac{n}{2} \leq p < n$

$$f(SE\min,n,p)=\frac{\left(6n+84p+53np-9n^2+n^3+26p^2-6p^3+23np^2-8n^2p+16\right)(p-n+1)(p-n)}{120}$$

Proposition 14. *The vulnerability* $F(SE\min, n, p)$ *of the successive elimination voting rule under maximin to strategic candidacy by opting out is as follows :*

For $0 < p \leq \frac{n}{2}$

$$F(SE\min,n,p)=\frac{\left(76p-70n-245np+100n^2+154p^2+26p^3-45np^2+20n^2p-16\right)(p+1)p}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

For $\frac{n}{2} \leq p < n$

$$F(SE\min,n,p)=\frac{\left(6n+84p+53np-9n^2+n^3+26p^2-6p^3+23np^2-8n^2p+16\right)(p-n+1)(p-n)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

Proposition 15. *When* n *tends to infinity, the vulnerability* $F'(SE\min, \alpha)$ *of the successive elimination voting rule under maximin to strategic candidacy by opting out is as follows:*

$$F'(SE\min,\alpha) = \left\{\begin{array}{l} \alpha^2\left(6\alpha^3+5\alpha^2-20\alpha+10\right) \text{ if } 0 \leq \alpha \leq \frac{1}{2} \\ (1-\alpha)^3\left(26\alpha^2-7\alpha+1\right) \text{ if } \frac{1}{2} \leq \alpha \leq 1 \end{array}\right.$$

Proposition 16. *When $\alpha = \frac{1}{2}$, the vulnerability $F''(SE\min, n)$ of the successive elimination rule under maximin to strategic candidacy by opting out is as follows:*

$$F''(SE\min,n) = \begin{cases} \frac{48n+75n^2+8n^3-75}{416n+240n^2+16n^3-1920} & \text{(if } n \text{ is odd)} \\ \frac{8n+24n^2+12n^3+n^4}{118n^2-34n+34n^3+2n^4-120} & \text{(if } n \text{ is even)} \end{cases}$$

4.1.3. *Successive elimination voting procedure: maximax behavior.*

Proposition 17. *Let $SE\max$ be the successive elimination voting procedure under maximax and consider α such that $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy by opting out if and only if*

$$\begin{cases} n_3+n_4+n_5+n_6 \geq \alpha n \\ n_1+n_3+n_4 > n-\alpha n \\ n_1+n_2+n_5 > n-\alpha n \\ n_1+n_2 \geq 1 \\ n_3+n_4 \geq 1 \\ n_5 \geq 1 \end{cases} \quad or \quad \begin{cases} n_3+n_4+n_5+n_6 \geq \alpha n \\ n_2+n_5+n_6 \geq \alpha n \\ n_1+n_2+n_3 > n-\alpha n \\ n_1+n_2 \geq 1 \\ n_5+n_6 \geq 1 \\ n_3 \geq 1 \end{cases}$$

Proposition 18. *Let $f(SE\max,n,p)$ be the total number of voting situations at which SE*max *is vulnerable to strategic candidacy. Computations give:*

For $0 < p \leq \frac{n}{2}$

$$f(SE\max,n,p)=\frac{p\left(55p-20n-165np+45n^2+15n^3+175p^2+65p^3+3p^4-25np^2-30n^2p+10np^3+5n^3p-15n^2p^2+2\right)}{60}$$

For $\frac{n}{2} \leq p < n$

$$f(SE\max,n,p)=\frac{\left(97p-27n-26np+8n^2+3n^3+38p^2-13p^3+24np^2-14n^2p+58\right)(p-n+1)(p-n)}{60}$$

Proposition 19. *The vulnerability $F(SE\max,n,p)$ of the successive elimination voting procedure under maximax to strategic candidacy by opting out is as follows:*

For $0 < p \leq \frac{n}{2}$

$$F(SE\max,n,p)=\frac{2p\left(55p-20n-165np+45n^2+15n^3+175p^2+65p^3+3p^4-25np^2-30n^2p+10np^3+5n^3p-15n^2p^2+2\right)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

For $\frac{n}{2} \leq p < n$

$$F(SE\max,n,p)=\frac{2\left(97p-27n-26np+8n^2+3n^3+38p^2-13p^3+24np^2-14n^2p+58\right)(p-n+1)(p-n)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

Proposition 20. *When n tends to infinity, the vulnerability $F'(SE\max,\alpha)$ of the successive elimination voting procedure under maximax to strategic candidacy by opting out is as follows:*

$$F'(SE\max,\alpha)=\begin{cases} 2\alpha^3\left(13\alpha^2-15\alpha+5\right) \text{ if } 0\leq\alpha\leq\frac{1}{2} \\ 2\left(1-\alpha\right)^2\left(-3\alpha^3+19\alpha^2-14\alpha+3\right) \text{ if } \frac{1}{2}\leq\alpha\leq 1 \end{cases}$$

Proposition 21. *When $\alpha=\frac{1}{2}$, the vulnerability $F''(SE\max,n)$ of the successive elimination voting rule under maximax to strategic candidacy by opting out is as follows:*

$$F''(SE\max,n)=\begin{cases} \frac{20n+3n^2+33}{160n+16n^2-384} \text{ (if } n \text{ is odd)} \\ \frac{464n+172n^2+36n^3+3n^4}{944n^2-272n+272n^3+16n^4-960} \text{ (if } n \text{ is even)} \end{cases}$$

4.2. **Opting in.**

4.2.1. *Amendment voting procedure.*

Proposition 22. *Let AmP be the amendment voting procedure and consider α such that $0<\alpha\leq 1$, then a voting situation s is vulnerable to strategic candidacy by opting in if and only if*

$$\begin{cases} n_1+n_2+n_5>n-\alpha n \\ n_4+n_5+n_6\geq\alpha n \\ n_1+n_3+n_4>n-\alpha n \\ n_3+n_4\geq 1 \\ n_5+n_6\geq 1 \\ n_2\geq 1 \end{cases} \quad or \quad \begin{cases} n_3+n_4+n_6\geq\alpha n \\ n_2+n_5+n_6\geq\alpha n \\ n_1+n_2+n_3>n-\alpha n \\ n_1+n_2\geq 1 \\ n_5+n_6\geq 1 \\ n_4\geq 1 \end{cases}$$

Proposition 23. *Let $f'(AmP,n,p)$ be the total number of voting situations at which AmP is vulnerable to strategic candidacy. Computations give*

For $0<p\leq\frac{n}{2}$

$$f'(AmP,n,p)=\frac{\left(29p-15n-35np+10n^2+32p^2-1\right)(p+2)(p+1)p}{120}$$

For $\frac{n}{2}\leq p<n$

$$f'(AmP,n,p)=\frac{\left(151p-61n-37np+6n^2+7n^3+61p^2-32p^3+61np^2-36n^2p+60\right)(p-n+2)(p-n+1)}{120}$$

Consequently,

Proposition 24. *The vulnerability $F'(AmP,n,p)$ of the amendment voting rule to strategic candidacy by opting in is as follows:*

For $0 < p \leq \frac{n}{2}$

$$F'(AmP,n,p)=\frac{\left(29p-15n-35np+10n^2+32p^2-1\right)(p+2)(p+1)p}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

For $\frac{n}{2} \leq p < n$

$$F'(AmP,n,p)=\frac{\left(151p-61n-37np+6n^2+7n^3+61p^2-32p^3+61np^2-36n^2p+60\right)(p-n+2)(p-n+1)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

Proposition 25. *When n tends to infinity, the vulnerability* $F'(AmP,\alpha)$ *of the amendment voting rule to strategic candidacy by opting in is as follows:*

$$F'(AmP,\alpha) = \begin{cases} \alpha^3\left(32\alpha^2 - 35\alpha + 10\right) \text{ if } 0 \leq \alpha \leq \frac{1}{2} \\ (1-\alpha)^3\left(32\alpha^2 - 29\alpha + 7\right) \text{ if } \frac{1}{2} \leq \alpha \leq 1 \end{cases}$$

Proposition 26. *When* $\alpha = \frac{1}{2}$*, the vulnerability* $F''(AmP,n)$ *of the amendment voting rule to strategic candidacy by opting in is as follows:*

$$F''(AmP,n) = \begin{cases} \frac{n^2-9}{160n+16n^2-384} \text{ (if } n \text{ is odd)} \\ \frac{8n+14n^2+7n^3+n^4-960}{944n^2-272n+272n^3+16n^4-960} \text{ (if } n \text{ is even)} \end{cases}$$

4.2.2. *Successive elimination voting procedure: maximin behavior.*

Proposition 27. *Let* $SE\min$ *be the successive elimination voting procedure under maximin and consider* α *such that* $0 < \alpha \leq 1$*, then a voting situation s is vulnerable to strategic candidacy by opting in if and only if*

$$\begin{cases} n_1 + n_2 + n_3 + n_5 > n - \alpha n \\ n_4 + n_5 + n_6 \geq \alpha n \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \end{cases} \quad or \quad \begin{cases} n_1 + n_2 + n_3 + n_5 > n - \alpha n \\ n_1 + n_2 + n_3 > n - \alpha n \\ n_3 + n_4 + n_6 \geq \alpha n \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \end{cases}$$

Proposition 28. *Let* $f'(SE\min,n,p)$ *be the total number of voting situations at which* $SE\min$ *is vulnerable to strategic candidacy. Computations give*

For $0 < p \leq \frac{n}{2}$

$$f'(SE\min,n,p)=\frac{\left(50n-59p-175np+80n^2+101p^2+26p^3-45np^2+20n^2p-14\right)(p-1)p}{120}$$

For $\frac{n}{2} \leq p < n$

$$f'(SE\min,n,p)=\frac{\left(37np-51p-9n-6n^2+n^3-11p^2-6p^3+23np^2-8n^2p+14\right)(p-n-1)(p-n)}{120}$$

Proposition 29. *The vulnerability $F(SE\min, n, p)$ of the successive elimination voting rule under maximin to strategic candidacy by opting in is as follows:*
For $0 < p \leq \frac{n}{2}$

$$F(SE\min,n,p)=\frac{\left(50n-59p-175np+80n^2+101p^2+26p^3-45np^2+20n^2p-14\right)(p-1)p}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

For $\frac{n}{2} \leq p < n$

$$F(SE\min,n,p)=\frac{\left(37np-51p-9n-6n^2+n^3-11p^2-6p^3+23np^2-8n^2p+14\right)(p-n-1)(p-n)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

Proposition 30. *When n tends to infinity, the vulnerability $F'(SE\min, \alpha)$ of the successive elimination voting rule under maximin to strategic candidacy by opting in is as follows:*

$$F'(SE\min,\alpha) = \begin{cases} \alpha^2\left(6\alpha^3+5\alpha^2-20\alpha+10\right) \text{ if } 0 \leq \alpha \leq \frac{1}{2} \\ (1-\alpha)^3\left(26\alpha^2-7\alpha+1\right) \text{ if } \frac{1}{2} \leq \alpha \leq 1 \end{cases}$$

Proposition 31. *When $\alpha = \frac{1}{2}$, the vulnerability $F''(SE\min, n)$ of the successive elimination voting procedure under maximin to strategic candidacy by opting in is as follows:*

$$F''(SE\min,n) = \begin{cases} \frac{25n^2-36n+4n^3-225}{208n+120n^2+8n^3-960} \text{ (if } n \text{ is odd)} \\ \frac{82n^2-56n+71n^3+8n^4}{944n^2-272n+272n^3+16n^4-960} \text{ (if } n \text{ is even)} \end{cases}$$

4.2.3. *Successive elimination voting procedure: maximax behavior.*

Proposition 32. *Let $SE\max$ be the successive elimination voting procedure under maximax and consider α such that $0 < \alpha \leq 1$, then a voting situation s is vulnerable to strategic candidacy by opting in if and only if*

$$\begin{cases} n_3+n_4+n_5+n_6 \geq \alpha n \\ n_1+n_3+n_4 > n-\alpha n \\ n_1+n_2+n_5 > n-\alpha n \\ n_1+n_2 \geq 1 \\ n_3+n_4 \geq 1 \\ n_6 \geq 1 \end{cases} \text{ or } \begin{cases} n_3+n_4+n_5+n_6 \geq \alpha n \\ n_2+n_5+n_6 \geq \alpha n \\ n_1+n_2+n_3 > n-\alpha n \\ n_1+n_2 \geq 1 \\ n_5+n_6 \geq 1 \\ n_4 \geq 1 \end{cases}$$

Proposition 33. *Let $f'(SE\max, n, p)$ be the total number of voting situations at which SE*max *is vulnerable to strategic candidacy. Computations give:*

For $0 < p \leq \frac{n}{2}$

$$f'(SE\max,n,p)=\frac{p\left(230p-85n-120np+15n^3+235p^2+70p^3+3p^4-5np^2-45n^2p+10np^3+5n^3p-15n^2p^2+62\right)}{60}$$

For $\frac{n}{2} \leq p < n$

$$f'(SE\max,n,p)=\frac{\left(74p-29n-18np+4n^2+3n^3+29p^2-13p^3+24np^2-14n^2p+30\right)(p-n+2)(p-n+1)}{60}$$

Proposition 34. *The vulnerability $F(SE\max, n, p)$ of the successive elimination voting procedure under maximax to strategic candidacy by opting in is as follows:*

For $0 < p \leq \frac{n}{2}$

$$F(SE\max,n,p)=\frac{2p\left(230p-85n-120np+15n^3+235p^2+70p^3+3p^4-5np^2-45n^2p+10np^3+5n^3p-15n^2p^2+62\right)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

For $\frac{n}{2} \leq p < n$

$$F(SE\max,n,p)=\frac{2\left(74p-29n-18np+4n^2+3n^3+29p^2-13p^3+24np^2-14n^2p+30\right)(p-n+2)(p-n+1)}{(n+5)(n-1)(n-2)(n+12)(n+1)}$$

Proposition 35. *When n tends to infinity, the vulnerability $F'(SE\max, \alpha)$ of the successive elimination voting procedure under maximax to strategic candidacy by opting in is as follows:*

$$F'(SE\max,\alpha) = \begin{cases} 2\alpha^3\left(13\alpha^2 - 15\alpha + 5\right) \; \textit{if } 0 \leq \alpha \leq \frac{1}{2} \\ (-2)\left(14\alpha - 19\alpha^2 + 3\alpha^3 - 3\right)(\alpha - 1)^2 \; \textit{if } \frac{1}{2} \leq \alpha \leq 1 \end{cases}$$

Proposition 36. *When $\alpha = \frac{1}{2}$, the vulnerability $F''(SE\max, n)$ of the successive elimination voting rule under maximax to strategic candidacy by opting in is as follows:*

$$F''(SE\max,n) = \begin{cases} \frac{5n^2-27n+3n^3-45}{416n+240n^2+16n^3-1920} \; \textit{(if n is odd)} \\ \frac{6n^3-8n^2-16n+3n^4-960}{944n^2-272n+272n^3+16n^4-960} \; \textit{(if n is even)} \end{cases}$$

4.3. Table of frequencies.

Opting out				Opting in			
n	AP	SE min	SE max	n	AP	SE min	SE max
3	0.25	0.5	0.5	3	0	0	0
4	0.138889	0.333333	0.222222	4	0	0.222222	0
5	0.117647	0.372549	0.254902	5	0.019608	0.176471	0.039216
6	0.103896	0.34632	0.186147	6	0.021645	0.255411	0.034632
7	0.092105	0.366228	0.210526	7	0.026316	0.232456	0.057018
8	0.089133	0.361416	0.173382	8	0.029304	0.282051	0.052503
9	0.081633	0.372449	0.193878	9	0.030612	0.266764	0.069971
10	0.081267	0.374656	0.168044	10	0.033517	0.303489	0.065197
11	0.076087	0.380737	0.18599	11	0.033816	0.291667	0.080314
12	0.076512	0.385849	0.165775	12	0.036405	0.321061	0.075278
15	0.070513	0.396368	0.179487	15	0.038462	0.326923	0.096154
18	0.069659	0.410284	0.165164	18	0.04193	0.358797	0.097052
21	0.066986	0.414428	0.177033	21	0.043062	0.36106	0.112624
24	0.066867	0.426187	0.166917	24	0.045377	0.383308	0.111644
27	0.065385	0.427404	0.176923	27	0.046154	0.383654	0.124038
30	0.065447	0.437311	0.168872	30	0.047808	0.400536	0.122177
33	0.064516	0.43704	0.177419	33	0.048387	0.39983	0.132428
36	0.064625	0.445522	0.170638	36	0.049628	0.413316	0.130144
39	0.06399	0.444446	0.17806	39	0.050079	0.412018	0.138857
42	0.064105	0.45183	0.172158	42	0.051045	0.423178	0.136382
45	0.063647	0.450306	0.178703	45	0.051408	0.421542	0.143941
48	0.063756	0.456828	0.173457	48	0.5218	0.431021	0.141399
51	0.063411	0.455053	0.1793	51	0.052478	0.429196	0.148063
54	0.06351	0.460886	0.174569	54	0.05311	0.437409	0.14552
57	0.063241	0.458976	0.179842	57	0.05336	0.435484	0.151473
60	0.06333	0.464246	0.175526	60	0.053886	0.442712	0.148966
63	0.063115	0.462271	0.180328	63	0.054098	0.440743	0.154339
66	0.063194	0.467074	0.176357	66	0.054544	0.447186	0.15189
∞	0.0625	0.5	0.1875	∞	0.0625	0.5	0.1875

5. Concluding and remarks

The main information brought by this work is how frequent parliamentary voting procedures, and specifically amendment and successive elimination voting rules, are vulnerable to strategic candidacy. First, it appears that they are vulnerable for any quota α.

In particular, when $\alpha = \frac{1}{2}$, for large electorates the vulnerability is $6,25\%$ for amendment voting procedure, and 50% and $18,75\%$ for successive elimination voting procedure with maximin and with maximax, respectively. The amendment voting procedure is vulnerable to strategic candidacy only in the presence of a Condorcet cycle (Type 1 or Type 2).

It appears that successive elimination voting procedure ($\geq 18,75\%$) seems to be much more vulnerable to strategic candidacy than amendment voting rule ($6,25\%$). Besides, note that the frequency under maximin (50%) is much more significant than under maximax ($18,75\%$).

It is also worth noting that the vulnerability with respect to the number of voters is a decreasing function for opting out and an increasing one for opting in.

As a future work, it would be interesting to study the vulnerability of positional rules (plurality, anti-plurality, Borda) to strategic candidacy. This would lead to see whether the frequencies are rather significant or similar to those of parliamentary voting procedures.

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